Calculus Warm Up # 7-3

1. Find the work needed to empty a full water tank over the top edge if it measures 4ft X 4ft X 4ft.

$= 7987.2$ ft-lbs

(integrate with calculator for #2 & 3)

2. Find the surface area of $y=x^2+2$ from $x=2$ to $x=4$ rotated around the y-axis

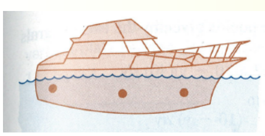
≈ 237.7

3. Find the arc length of $y = x^2+2$ from $x=2$ to $x=4$

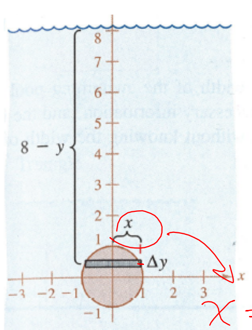
≈ 12.17

Friday space:

A circular observation window on a ship has a radius of 1 foot and is centered 8 feet below water level. What is the fluid force on the window?



weight of sea water = 64 $\frac{\text{lbs}}{\text{ft}^3}$



Use calculator to integrate.

$F = (\text{pressure})(\text{area})$

$F = \int_{-1}^{1} (64)(8-y) \cdot 2\sqrt{1-y^2}\, dy$

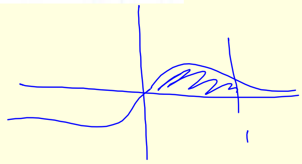
$x = \sqrt{1-y^2}$

HW Questions: p. 354

In Exercises 1–14, sketch the region bounded by the graphs of the given equations and determine the area of the region.

1. $y = \frac{1}{x^2}$, $y = 0$, $x = 1$, $x = 5$

3. $y = \frac{x}{(x^2+1)^2}$, $y = 0$, $x = 1$



$$A = \frac{1}{2}\int_0^1 \frac{2x}{(x^2+1)^2}\,dx$$

$u = x^2 + 1$

$du = 2x\,dx$

5. $x = y^2 - 2y$, $x = 0$

7. $y = x$, $y = x^3$

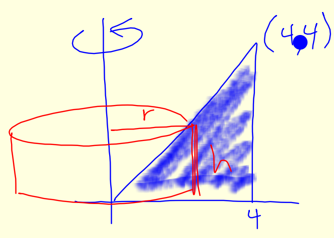
9. $y = x^2 - 8x + 3,\ y = 3 + 8x - x^2$

In Exercises 15–22, find the volume of the solid generated by revolving the plane region bounded by the given equations about the indicated line.

15. $y = x,\ y = 0,\ x = 4$

(a) the x-axis (b) the y-axis Shells

(c) the line $x = 4$ (d) the line $x = 6$



$r = x \quad h = x$

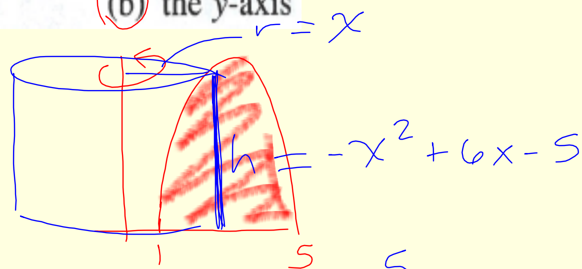
$$V = 2\pi \int_0^4 x^2\,dx$$

$$\boxed{V = \frac{128\pi}{3}}$$

In Exercises 15–22, find the volume of the solid generated by revolving the plane region bounded by the given equations about the indicated line.

21. $y = -x^2 + 6x - 5$, $y = 0$

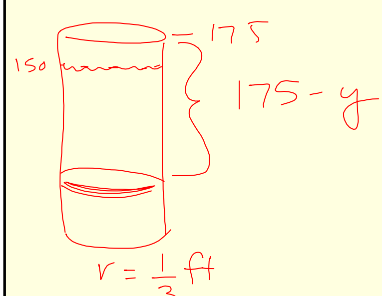
(a) the x-axis (b) the y-axis



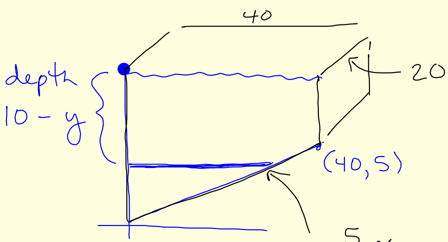
$$V = 2\pi \int_1^5 x(-x^2+6x-5)\,dx$$

23. Find the work done in stretching a spring from its natural length of 10 inches to a length of 15 inches, if a force of 4 pounds is needed to stretch it 1 inch from its natural position.

25. A water well has an 8-inch casing (diameter) and is 175 feet deep. If the water is 25 feet from the top of the well, determine the amount of work done in pumping it dry, assuming that no water enters the well while it is being pumped.



29. A swimming pool is 5 feet deep at one end and 10 feet deep at the other, and the bottom is an inclined plane. The length and width of the pool are 40 feet and 20 feet, respectively. If the pool is full of water, what is the fluid force on each of the vertical walls?



$y = \frac{5}{40}x$

$x = 8y$

$L(y) = \begin{cases} 8y, & 0 \le y \le 5 \\ 40, & 5 \le y \le 10 \end{cases} \rightarrow \int_0^5 + \int_5^{10}$

Deep End Wall

$F = 62.4\int_0^{10}(10-y)(20)\,dy$

Shallow end

$F = 62.4\int_5^{10}(10-y)(20)\,dy$

Side Walls

$F = 62.4\int_0^5 (10-y)(8y)\,dy$

$+\ 62.4\int_5^{10}(10-y)(40)\,dy$

38. Find the length of the graph of

$$y = \frac{1}{6}x^3 + \frac{1}{2x}$$

from $x = 1$ to $x = 3$.

$y' = \frac{x^2}{2} - \frac{1}{2x^2}$ $\quad (y')^2 = \frac{x^4 - 2 + x^{-4}}{4}$

$S = \int_1^3 \sqrt{1 + \frac{x^4 - 2 + x^{-4}}{4}}\,dx$

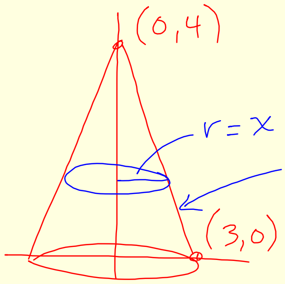
$= \frac{1}{2}\int_1^3 \sqrt{x^4 + 2 + x^{-4}}\,dx$

$= \frac{1}{2}\int_1^3 \sqrt{(x^2 + x^{-2})}\,dx$

$= \frac{1}{2}\int_1^3 (x^2 + x^{-2})\,dx$

$= \boxed{\frac{14}{3}}$

39. Use integration to find the lateral surface area of a right circular cone of height 4 and radius 3.



$y = -\frac{4}{3}x$

$\left(y' = -\frac{4}{3}\right)^2$

$(y')^2 = \frac{16}{9}$

$$S = 2\pi \int_0^3 x\sqrt{1+\frac{16}{9}}\,dx$$

$$= 2\pi \int_0^3 x\sqrt{\frac{9+16}{9}}\,dx$$

$$= \frac{5}{3} \cdot 2\pi \int_0^3 x\,dx$$

$$= \boxed{15\pi}$$

HW: Yellow WS

* Answers follow.

:)

Yellow Ch. 6 Rev. Part II

1a) $\frac{4}{5}$

b) $\frac{1}{4}$

2a) $\frac{512\pi}{15}$

b) 64π

3a) 8π

b) $\frac{32\pi}{5}$

c) $\frac{8\pi}{3}$

d) $\frac{176\pi}{15}$

4a) $\frac{512}{15}$

b) $\frac{128\sqrt{3}}{15}$

c) $\frac{64\pi}{15}$

5) 7020π ft-lbs

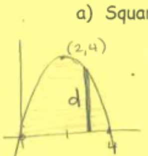
6) $17{,}333.\overline{3}\,\pi$ ft-lbs.

✯ worked out solutions for #4-6 are next. :)

$0 = x(4-x)$

4. Sketch the base bounded by: $y = 4x - x^2$ and the x-axis. Find the volume of the solid formed by taking cross sections perpendicular to the x-axis with the given shapes:

a) Square $V = s^2 \Delta x$



$s = d = 4x - x^2$

$r = \frac{1}{2}(4x - x^2)$

$V = \int_0^4 (4x - x^2)^2 dx$

$V = \boxed{\frac{512}{15}}$

b) Equilateral Triangle

$V = \frac{\sqrt{3}}{4} s^2 \Delta x$

$V = \frac{\sqrt{3}}{4} \int_0^4 (4x - x^2)^2 dx$

$= \boxed{\frac{128\sqrt{3}}{15}}$

c) Semicircle

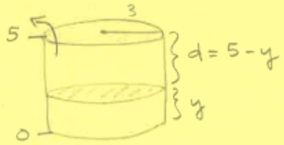
$V = \frac{1}{2}\pi r^2 \Delta x$

$V = \frac{1}{2}\pi \int_0^4 \left[\frac{1}{2}(4x - x^2)\right]^2 dx$

$V = \frac{\pi}{8} \int_0^4 (4x - x^2)^2 dx$

$= \boxed{\frac{64}{15}\pi}$

5. A cylindrical tank of radius 3 feet is 5 feet high and full of water. How much work is done pumping out all the water through a hole in the top of the tank? (The weight of the water is 62.4 pounds per cubic foot).



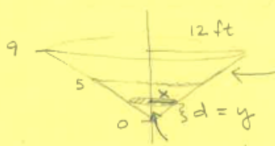
$$W = \int_0^5 \underbrace{(5-y)}_{d}\underbrace{(561.6\pi)}_{\Delta F}\,dy$$

$$W = \boxed{7020\pi \text{ ft}\cdot\text{lbs}}$$

$$\Delta F = \left(\frac{62.4 \text{ lbs}}{\cancel{\text{ft}^3}}\right)\left(\pi(3)^2 \Delta y \ \cancel{\text{ft}^3}\right)$$

$$= 561.6\pi \text{ lbs}$$

6. A 9 ft. tall conical tank with a radius of 12 ft. is empty. How much work would be done filling the tank with water to a level 4 feet from the top of the tank from a hole at the bottom of the tank?



$y = \frac{3}{4}x$

$x = \frac{4}{3}y$

$$\Delta F = \left(\frac{62.4 \text{ lbs}}{\text{ft}^3}\right)\left(\pi\left(\frac{4}{3}y\right)^2 \Delta y \text{ ft}^3\right)$$

$$= \frac{1664}{15}\pi y \text{ lbs.}$$

$$W = \int_0^5 y\left(\frac{1664}{15}\pi y^2\right)dy$$

$$= \frac{1664}{15}\pi \int_0^5 y^3\,dy$$

$$\boxed{17333.\overline{3} \quad \pi \text{ ft}\cdot\text{lbs}}$$